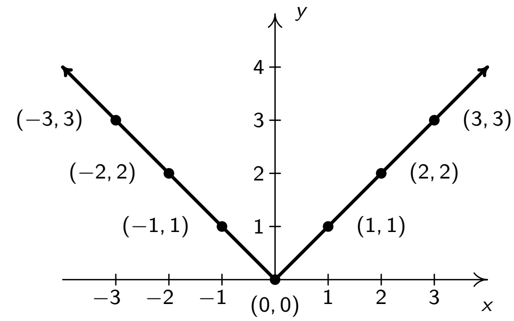
MATH 1650: ABSOLUTE VALUE FUNCTIONS

EXAMPLE: Graph $f(x) = |x|$ set of axes below.

x	$f(x) = \|x\|$	$(x, f(x))$
-3	3	$(-3, 3)$
-2	2	$(-2, 2)$
-1	1	$(-1, 1)$
0	0	$(0, 0)$
1	1	$(1, 1)$
2	2	$(2, 2)$
3	3	$(3, 3)$



$f(x) = |x|$

EXAMPLE: Graph each of the following functions using a graphing utility and record the information requested.

- $f(x) = |x - 3| + 2$
  - vertex: $(3, 2)$
  - opens: up
  - slopes of lines: ± 1
- $f(x) = -2|x + 1| + 4$
  - vertex: $(-1, 4)$
  - opens: down
  - slopes of lines: ± 2
- $f(x) = \dfrac{1}{2}|x - 1| - 2$
  - vertex: $(1, -2)$
  - opens: up
  - slopes of lines: $\pm \dfrac{1}{2}$

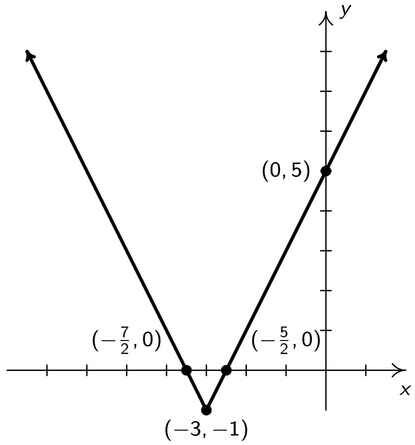
EXAMPLE: Consider: $f(x) = 2|x + 3| - 1$.

- Vertex: $(-3, -1)$
- $f(0) = 2|(0) + 3| - 1 = 2|3| - 1 = 5$ so the y-intercept is $(0, 5)$
- Solving $f(x) = 0$ means solving $2|x + 3| - 1 = 0$:

$$\begin{aligned} 2|x+3| - 1 &= 0 && \text{add 1 to both sides} \\ 2|x+3| &= 1 && \text{divide both sides by 2} \\ |x+3| &= \frac{1}{2} && \text{drop the absolute values} \\ x + 3 &= \pm\frac{1}{2} \end{aligned}$$

We get $x = -\dfrac{5}{2}$ or $x = -\dfrac{7}{2}$ which correspond to the x-intercepts: $\left(-\dfrac{5}{2}, 0\right)$ and $\left(-\dfrac{7}{2}, 0\right)$

- Graph of $f(x)$:



$f(x) = 2|x+3| - 1$

the **domain** of f: $(-\infty, \infty)$

the **range** of f: $[-1, \infty)$

the **maximum** of f: none.

the **minimum** of f: -1

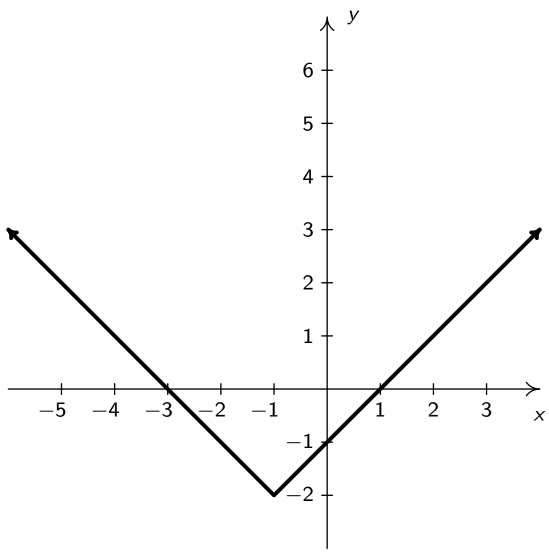
f is:

increasing: on $[-3, \infty)$

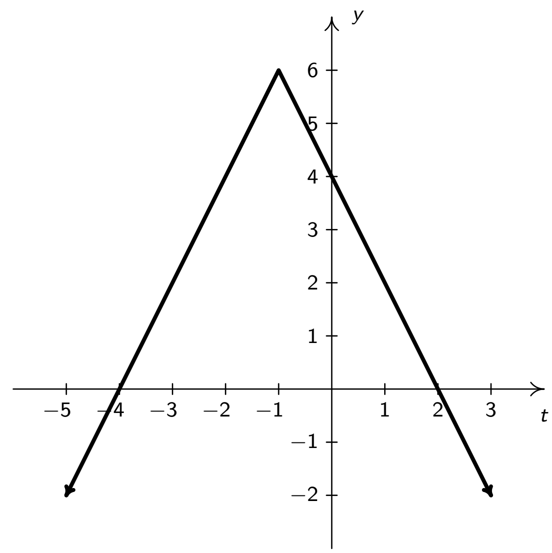
decreasing: on $(-\infty - 3]$

constant: nowhere

EXAMPLE: Find a possible formula for each function whose graph appears below:



$y = f(x) = |x+1| - 2$



$y = G(t) = -2|t+1| + 6$

EXAMPLE: Rewrite each of the functions below in the form $f(x) = a|x-h| + k$. Identify the vertex.

- $f(x) = |2x+3| - 4 = 2\left|x + \dfrac{3}{2}\right| - 4$
- vertex: $\left(-\dfrac{3}{2}, -4\right)$
- $f(x) = |5-x| + 1 = |x-5| + 1$
- vertex: $(5, 1)$
- $f(x) = \dfrac{|x-3|+1}{2} = \dfrac{1}{2}|x-3| + \dfrac{1}{2}$
- vertex: $\left(3, \dfrac{1}{2}\right)$
- $f(x) = \left|\dfrac{5-2x}{3}\right| - 4 = \dfrac{2}{3}\left|x - \dfrac{5}{2}\right| - 4$
- vertex: $\left(\dfrac{5}{2}, -4\right)$

DEFINING THE ABSOLUTE VALUE AS A PIECEWISE-DEFINED FUNCTION:

EXAMPLE: Consider $f(x) = |x-3| - x + 2$. $f(x)$ **cannot** be written in the form $f(x) = a|x-h| + k$. Why? There is an 'x' both inside the absolute values and outside. Hence, we can't write it as $f(x) = a|x-h| + k$.

EXAMPLE: Let $f(x) = |2x-3| + x - 4$. Write $f(x)$ as a piecewise-defined function.

1. Write $|2x-3|$ as a piecewise-defined function.

   Since $|x| = \begin{cases} -x, & \text{if } x < 0 \\ x, & \text{if } x \geq 0 \end{cases}$, we replace '$x$' with '$(2x-3)$' to get a formula for $|2x-3|$:

$$|2x-3| = \begin{cases} -(2x-3), & \text{if } (2x-3) < 0 \\ (2x-3), & \text{if } (2x-3) \geq 0 \end{cases},$$

   - Simplify: $-(2x-3) = -2x+3$
   - Simplify: $(2x-3) = 2x-3$
   - Solve: $(2x-3) < 0$: $x < \dfrac{3}{2}$
   - Solve: $(2x-3) \geq 0$: $x \geq \dfrac{3}{2}$

   Hence,

$$|2x-3| = \begin{cases} -2x+3, & \text{if } x < \dfrac{3}{2} \\ \\ 2x-3, & \text{if } x \geq \dfrac{3}{2} \end{cases}$$

2. Next, we use the pieces from $|2x-3|$ to break $f(x)$ into pieces:

   - If $x < \dfrac{3}{2}$, $|2x-3| = -2x+3$, so $f(x) = |2x-3| + x - 4 = -2x + 3 + x - 4 = -x - 1$
   - If $x \geq \dfrac{3}{2}$, $|2x-3| = 2x-3$, so $f(x) = |2x-3| + x - 4 = 2x - 3 + x - 4 = 3x - 7$

   Putting it all together, we get

$$f(x) = \begin{cases} -x-1, & \text{if } x < \dfrac{3}{2} \\ \\ 3x-7, & \text{if } x \geq \dfrac{3}{2} \end{cases}$$